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This report results from a contract tasking Bauman State Technical University as follows: Provide methods for structural analysis of random loading given by correlation functions or spectral densities making it possible to obtain initial information about loading cycles which is necessary for assessing survivability and fatigue longevity of cracked structural elements.

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Final report on special contract SPC-95-4011

FAIL-SAFE PREDICTION IN STRUCTURES UNDER RANDOM LOADS WITH COMPLICATED TRAJECTORIES

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Annotation

Methods for structural analysis of random loading given by correlation functions or spectral densities making it possible to obtain initial information about loading cycles which is necessary for assessing survivability and fatigue longevity of cracked structural elements are considered. A new method to analyze random loading of complex structure based on substituting the given process of loading with simple structure equivalent to the given one by the effect of damage, which gives an opportunity to use in longevity analysis standard information about mechanical properties of material is developed.

Key words :

Random processes, strength, fatigue, crack resistance, survivability.

1. Introduction.

To assess reliability characteristics of a structure under random loading one has to obtain the values of probability that the parameters describing loading and the working state of the structure are within legitimate borders in the preassigned period of time. Here principally important are stresses, fatigue damage accumulated, fatigue crack length, stress intensity factors, etc. The structure abilities in the terms of resistance are determined by extreme allowable values of these parameters and can by explicitly shown as the result of standard tests on cyclic loading effects and can be represented as relationships between mechanical properties and parameters of loading cycle (amplitudes and mean values).

As an example, on **Fig.1** is given the principle characteristic of fatigue strength of structures - the schematized fatigue surface showing number of cycles of loading to destruction **N** starting from the level of amplitudinal σ_a and mean σ_m stresses **[1,2]**.

On **Fig.2** is stretched the basic characteristic of cyclic crack resistance of the structural materials - full schematized cracked structural element survivability diagram.

Here **1** is the area where fatigue crack propagation rates are infinitesimal; **2** is the area with moderate rates of fatigue crack propagation (from 10^{-7} to 10^{-3} mm per cycle); **3** is the area with exceedingly high rates of fatigue crack propagation (more than 10^{-3} mm per cycle). Here K_p and K_m symbolize peak-to-peak and mean value of **SIF** in given loading cycles respectively; K_{1c} - static viscosity of destruction; $K_{fc,o}$ and $K_{fc,-1}$ - cyclic viscosity of destruction in starting-from-zero and symmetrical cycles; $K_{th,o}$ and $K_{th,-1}$ - threshold values of **SIF** in the same kinds of cycles respectively when there is no crack growth.

The main problem, however, is that cycles of loading in random processes of complex structure and corresponding amplitudes and mean values of the cycles cannot be defined unambiguously. An example of such process is given in **Fig.3a**. It is hard for such process to use information analogous to one represented on **Fig.1,2**, whereas for the process shown on **Fig.3b** there is no such problem. Let us notice that for the former process the ratio of the number of extremums to the number of points where the curve crosses the mean level is approximately equal to **2** (the higher this ratio rises, the more complex structure of random process becomes) while for the latter one this ratio is sharply **1**. If it becomes possible to substitute a process of loading complex structure with a process of simple structure equivalent to the former one by the effect of damage, there appears an opportunity to use standard information about mechanical properties of materials shown on **Fig.1,2** in longevity analysis for random loading processes with complex structure. The complication is that, as a rule, random loading processes are given not by their time histories, but by probabilistic characteristics - correlation functions or energy spectra with the help of which it is required to bring out all possible cycles of loading.

For the strength analysis, there must be brought out the structure of random processes using statistical information at the hand - correlation functions or energy spectra of the processes. While in terms of statistics there must be defined : number of points where trajectory of the process crosses the **0** - level (the mean level) number of exceeding over arbitrarily chosen level, maximums, minimums, supremum, time intervals between zeroes, extremums, etc. At the same time, an important problem is to define all the cycles that constitute the overall process of loading.

2. Random process structure analysis and loading description.

Let us consider the Gaussian stationary random process **x(t)** with known correlation function :

$$K_x(\tau) = \langle x(t) \cdot x(t+\tau) \rangle \text{, (1)}$$

which characterizes how heavily the structure is loaded. In this case, mutual correlation function for **ν**-th and **l**-th derivatives of **x(t)** can be determined as follows :

$$K_{x^{(\nu)}x^{(l)}}(\tau) = (-1)^{\nu} K_x^{(\nu+l)}(\tau) \text{, } (\nu, l = 0,1,2,\ldots) \text{, (2)}$$

From **(2)** at **τ=0** we obtain correlation moments of mutual distribution of probability for the process **x(t)** and its derivatives. therefore let us assume that density $f(x, x^{(1)}, x^{(2)} \ldots)$ of this distribution is known. The problem of the process analysis consists in determining probabilistic characteristics of the number of zeroes, extremums and random process values at special points of trajectory where **n**-th derivative of **x(t)** equals zero as well as in complete bringing out of all its cycles.

The functional necessary for defining the number of trajectory special points at which **n**-th derivative is equal to zero in time **t** may be written :

$$N_{(n)}(t) = \int_0^t \left| x^{(n+1)}(\tau) \right| \cdot \delta(x^{(n)}(\tau)) d\tau \text{, (3)}$$

where **δ** is Dirac delta function.

From **(3)** one obtains the following expression to gain the mean number of these points per unite of time :

$$\langle N_{(n)} \rangle = \int_{-\infty}^{\infty} \left| x^{(n+1)} \right| \cdot f(0, x^{(n+1)}) dx \text{, (4)}$$

where $f(0,x^{(n+1)})$ is mutual density of probability distribution for n-th and $(n+1)$-th derivatives for $x^{(n)}=0$, which for the Gaussian stationary process is represented by following correlation moment matrix :

$$[M]=(-1)^{2n+1}\begin{bmatrix} K_x^{(2n)}(0) & K_x^{(2n+1)}(0) \\ K_x^{(2n+1)}(0) & K_x^{(2n)}(0) \end{bmatrix}, \quad (5)$$

Taking (5) into account, one can get from (4)

$$\langle N_{(n)} \rangle = 1/\pi[-K_x^{(2n+2)}(0)/K_x^{(2n)}(0)]^{1/2} \qquad (6)$$

From (6) it is possible to acquire the sequence of the numbers $k_1=\left\langle\frac{N_1}{N_0}\right\rangle$, $k_2=\left\langle\frac{N_2}{N_1}\right\rangle$, ... , $k_n=\left\langle\frac{N_n}{N_{n-1}}\right\rangle$ which thoroughly characterizes random process structure complexity. Let us note that $k_1= k_2=\ldots k_n=1$, whereas if $n\to\infty$, $R_n\to 1$.For a random process with correlation function $K(\tau)=K_0\cos(\omega_0 t)$ one has $\langle N_0\rangle=\langle N_1\rangle=$, ... ,$=\langle N_n\rangle=\frac{\omega_0}{\pi}$ and $k_1= k_2=\ldots k_n=1$. If a random process has correlation function written as $K(\tau)=K_0\delta(\tau)$ (i.e. the process is Gaussian white noise),
then :

$$k_n=\frac{2n+1}{\sqrt{(2n-1)(2n+3)}}.$$

Thus it can be inferred that for Gaussian white noise $k_1=\frac{3}{\sqrt{5}}\approx 1.35$, $k_2=\frac{5}{\sqrt{21}}\approx 1.09$. More complex structure has e.g. a process with the following correlation function $K(\tau)=K_0e^{-\alpha^2t^2}$, for which $k_1=\sqrt{3}\approx 1.73$, $k_2=\sqrt{\frac{5}{3}}\approx 1.29$ for every α.

Since the probability that a special point (where n-th) derivative equals zero) is in square (dx,dt) can be obtained as the product of probabilities that the point is in the intervals dt and dx simultaneously, one can acquire the following expression for random process value probability distribution at special points :

$$f_n(x)=\frac{1}{\langle N_n\rangle}\int_{-\infty}^{\infty} x^{(n+1)}f(x,0,x^{(n+1)})dx^{(n+1)}, \quad (7)$$

where $f(x,0,x^{(n+1)})$ - density of mutual distribution of probability for $x,x^{(n)}$ and $x^{(n+1)}$ for $x^{(n)}=0$. The density for the Gaussian stationary process is determined by the matrix correlation moments as follows :

$$[M]=(-1)^{2n+1}\begin{bmatrix} K_x(0) & K_x^{(n+1)}(0) \\ K_x^{(n+1)}(0) & K_x^{(2n+2)}(0) \end{bmatrix}, \quad (8)$$

From (7) if $n=1$, one obtains probability distribution density for extremums, if $n=2$ - the same for random process values at inflection points, etc.

For further analysis, let us assume that random process values at inflection points coincide with mean values of cycles whereas amplitudes x_a and mean values of cycles x_m determined by two succession extremums are statistically independent. Thus one can find out the values of the following first and second moments of distribution [3] :

$$\langle x_a \rangle = \frac{1}{k_1}\left[\frac{\pi}{2}K_x(0)\right]^{\frac{1}{2}} , (9)$$

$$\langle x_m \rangle = 0 , (10)$$

For corresponding probability densities one has the following expressions when $K_x(0)=1$:

$$f_a(x) = k_1^2 x \exp\left(-\frac{k_1^2 x^2}{2}\right) , (11)$$

$$f_m(x) = \frac{1}{\sqrt{2\pi(1-k_1^{-2})}} \exp\left(-\frac{x^2}{2(1-k_1^{-2})}\right) , (12)$$

Note that equalities (11) and (12) depend on parameter k_1 only. Let us omit index 1 for k_1 in further investigations.

Let us now find out all cycles in the given random process $x(t)$ and form them a process with simple structure equivalent to the given one by the effects of damage. To determine probability density for amplitudes in the process formed one may use the procedure of gradual exclusion from the given process of cycles defined by to neighboring extremums which, with amplitudes growing, are step-by-step transferred into the formed process. On Fig.3,4 is shown gradual transformation of a complex process with complexity parameter $k \approx 2$ into a simple process with $k \approx 2$.

Let us denote $n_{o,o}$ to be mean numbers of zeroes per unit of time, $n_{e,o}$ - the mean number of extremums per unit of time, $f_a(x,k_o)$ - probability density for amplitudes of cycles in initial process with initial value $k_o=n_{e,o}/n_{o,o}$, where $n_{o,o}(x)$, $n_{e,o}(x)$, $k(x)$, $c(x)$, $f_a(x,k(x))$ are the mean number of zeroes per unit of time, the mean number of extremums per unit of time, the factor of trajectory complication , the factor of standardization and the probability density function for the amplitudes for initial process after exclusion from it of the cycles with amplitude values up to x accordingly.

For further we shall accept a hypothesis that the intensity of process of exclusion of extremums k times greater than the intensity of process of exclusion of zeroes which will be expressed by the correlation:

$$\frac{n_e'(x)}{n_e(x)} = k(x) \cdot \frac{n_o'(x)}{n_o(x)} , (13)$$

where

$$k(x) = \frac{n_e(x)}{n_o(x)} , (14)$$

Let us denote the required density of probability distribution for amplitudes in the forming equivalent new random process of simple structure for which $k=1$ by $f(x)$. As number of extremums excluded from initial process equals to number of extremums included in formed process the following equality is received **(Fig.5):**

$$-dn_e(x) = n_e(x)\cdot c(x)\cdot f_a(x,k(x))dx = n_{e,o}\cdot f(x)dx\ ,$$

where pursuant to **(11)** $\cdot c(x)\cdot f_a(x,k(x))dx = x\cdot k^2(\mathrm{x})$,

These equality gives the expressions :

$$f(x) = \frac{x\cdot k^2(x)\cdot n_e(x)}{n_{e,o}}\ ,\ (15)$$

$$n_e(x) = -x\cdot k^2(x)\cdot n_e(x)\ ,\ (16)$$

Thus one comes to four equations **(13) - (16)** to determine four unknown functions **f(x),k(x),$\mathbf{n_e(x)}$,$\mathbf{n_o(x)}$.**

From **(13)** and **(16)** one receives the expressions:

$$\frac{n_e'(x)}{n_e(x)} = -x\cdot k^2(x)\ ,\ (17)$$

$$\frac{n_o'(x)}{n_o(x)} = -x\cdot k^2(x)\ ,\ (18)$$

Integrating **(17)** and **(18)** gives:

$$n_e(x) = n_{e,o}\cdot \exp\left(-\int_0^x z\cdot k^2(z)dz\right),\ (19)$$

$$n_o(x) = n_{o,o}\cdot \exp\left(-\int_0^x z\cdot k^2(z)dz\right),\ (20)$$

From (19) and (20) excluding the functions $\mathbf{n_e(x)}$ and $\mathbf{n_o(x)}$ the following equality is obtained:

$$k(x)\cdot \exp\left(-\int_0^x z\cdot k(z)dz\right) = k_o\cdot \exp\left(-\int_0^x z\cdot k^2(z)dz\right),\ (21)$$

Taking the derivative **(21)** one comes to a differential equation for function **k(x)** :

$$\frac{dk}{(k^2(k-1))} = -x\cdot dx\ ,\ (22)$$

where for **x=0** we have **k=k$_\mathbf{o}$.**

Integrating the equation **(22)** gives the following algebraic equation to determine the function **k(x):**

$$\frac{1}{k}-\frac{1}{k_o}+\ln\left[\frac{k_o(k-1)}{k(k_o-1)}\right]=\frac{x^2}{2}, \quad (23)$$

The required functions $n_o(x)$ and $n_e(x)$ are received after substituting (22) in (19) and (20):

$$n_o(x)=\frac{n_{o,o}\cdot k_o(k(x)-1)}{k(x)\cdot(k_o-1)}, \quad (24)$$

$$n_o(x)=\frac{n_{o,o}(k(x)-1)}{\cdot k_o-1}, \quad (25)$$

Combining (25) and (15) one can find the required probability density function for the amplitudes of loading cycles as follows :

$$f(x)=\frac{x\cdot k^2(x)\cdot(k(x)-1)}{k_o-1}, \quad (26)$$

where function $k(x)$ is defined from the solution of the equation (23).

The curves representing the functions $f(x)$ and $f_a(x)$ are compared for $k_0=k_1=3$ in the Fig.6 . It is visible from them that in considered case these functions are essentially different. This distinction will increase at increase of factor k and it will decrease at it reduction . These curves will coincide for $k_1=k_0=1$.

From (23) substituting function $k(x)$ in (12) instead of k_1 the probability density function for mean values of cycles which is corresponds to the given level of amplitudes is received.

From (12) and (26) denoting the amplitude by x_a and the mean value of the cycle in the formed process by x_m one obtains the following expression for the mutual probability density function for these values :

$$f(x_a,x_m)=f(x_a)f_m(x_m,k(x_a)), \quad (27)$$

If process $x(t)$ represents a process of change of stresses then expression (27) may be directly used in calculations on fatigue life [3]. So for the mean fatigue damage accumulation under one loading cycle the following expression is received:

$$v=\int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{f(x_a,x_m)}{N(x_a,x_m)}dx_a dx_m$$

where $N(x_a,x_m)$ - number of loading cycles up to destruction determined in the **Fig.1**.

3. Estimation of crack stability and fail-safe of structures.

Consider an element of a structure with an initial crack having length l under the action of nominal stresses $\sigma(t)$ representing Gaussian stationary process complex by a structure. Pursuant to **(26)** and **(27)** such process can be consider as consisting of cycles with known probable characteristics.

Let us accept that the strength of such element is completely determined by a stresses intensity factor **(SIF)**:

$$K(t) = \sigma(t)\sqrt{\pi \cdot l(t)} \cdot f(\tilde{l})\ ,\ (29)$$

where **l(t)** - crack length to a time moment **t, f(l)** - undimension function of the relative crack length .

It is necessary to take into account the ranges $\mathbf{K_r}$ and the mean values $\mathbf{K_m}$ of the cycles in the process **K(t)** in accordance with features of representation of the information about fracture toughness of materials (for example, **Fig.2**). These parameters of cycles are determined from the formulas :

$$K_r = \begin{cases} 0\ ,\ K_{max} < 0 \\ K_{max}\ ,\ \geq 0\ ,\ K_{max} \leq 0 \\ K_{max} - K_{min}\ ,\ K_{max} > 0 \end{cases}\ ,\ (30)$$

$$K_m = \begin{cases} 0\ ,\ K_{max} < 0 \\ 0.5 \cdot (K_{max} + K_{min})\ ,\ K_{max} \geq 0 \end{cases}\ ,\ (31)$$

where $\mathbf{K_{max}},\mathbf{K_{min}}$ - maximum and minimum value of **SIF** for the loading cycles.

A coefficient of asymmetry for the cycles is defined as :

$$R = \frac{K_{min}}{K_{max}}$$

We shall result the cycles of the process **K(t)** with any values $\mathbf{K_r}$ and $\mathbf{K_m}$ to equal on speed of crack's growth equivalent pulsable cycles with R=0.

Corresponding ranges of the cycle will be denoted by $\mathbf{K_e}$, and the process of increase of the crack lengths shall be expressed by the following kinetics equation:

$$l = \begin{cases} 0\ ,\ K_e < K_{th} \\ \alpha \cdot K_e^n\ ,\ K_{th} \leq K_e \leq K_{fc} \\ \infty\ ,\ K_e > K_{fc} \end{cases}\ ,\ (32)$$

where the $\mathbf{K_{th}}$ - threshold of **SIF** under which growth of cracks not yet occurs,$\mathbf{K_{fc}}$ - cyclic fracture toughness, α and **n** - parameters.

To the dependence from the degree of asymmetry of loading cycles threshold of the **SIF** is denoted by $\mathbf{K_{th,-1}}$ - for the symmetric loading cycle and by $\mathbf{K_{th,o}}$ - for the pulsable loading cycle. Parameters of cyclic fracture toughness are denoted by $\mathbf{K_{fc,-1}}$ and $\mathbf{K_{fc,0}}$ similarly.

Cycles with parameters $\{K_r=K_{fc,-1},K_m=0\}$, $\{K_r=K_{fc,o},K_m=0.5K_{fc,o}\}$ and $\{K_r=0,K_m=K_{TC}\}$ as well as the cycles with parameters $\{K_r=K_{th},K_m=0,K_r=K_{th,o},\ K_m=0.5K_{th,o}\}$ and $\{K_r=0,K_m=K_{TC}\}$ are equivalent. In the first case there will be cycles with inadmissible high speeds of crack's growth and in the second - with the neglectly small speeds of crack's growth. In the system of coordinates (K_r, K_m) the determined point corresponds to the each of the mentioned above loading cycles. After connecting these points by direct lines we shall receive such combinations of sizes K_r and K_m on these lines at which speed of crack's growth will be inadmissible high and such their combination at which speed of crack's growth will be neglectly small. Thus the fail-safe diagram shown in a **Fig.2** is received. On this diagram it is possible to denote three zones: **1**-with non-propagating cracks, **2**- with cracks speed of growth of which are described by equation **(32)**, **3** - with inadmissible high speeds of crack's growth . Compressing stress in cycles of loading with R=0...-1 has insignificant influence on speed of crack's growth and it is possible to accept that $K_{fc,0}=K_{fc,-1}$ and $K_{th,o}=K_{th,-1}$. At the same time influence of the high compressing stress on the speed of crack's growth is investigated only a little and consequently for **R<-1** the fail-safe diagram has hypothetical character and is shown on a **Fig.2** by a shaped line.

We shall estimate the stock of the crack stability to provoke the growth and transition in the condition of gradual fatigue crack propagation as well as the stock of stability to transition in the condition of their uncontrollable growth causing the complete destruction of structure.

In accordance with the fail-safe diagram **(Fig.2)** the condition of crack stability to provoke the growth consists in a volume that the points describing cycles of loading will be in the area Ω_1 and condition of stability of structure elements with cracks to destruction (condition fracturetoughness) - that these points will be in the area $\Omega_1+\Omega_2$.

Let the given loading cycle with parameters K_r and K_m be represented on the fail-safe diagram by the point A_1 or A_2 (Fig. 4). Then in accordance with the sign of the coefficient of asymmetry of the cycles, the coefficient n_s of the stock of the crack's stability and coefficient **n** of the stock of the fracture toughens of the structure element with the crack can be determined from the formulas:

$$n_s = \begin{cases} \dfrac{OB_1}{OA_1}, & R \geq 0 \\ \dfrac{OB_2}{OA_2}, & R < 0 \end{cases}, \quad (33)$$

$$n_c = \begin{cases} \dfrac{OC_1}{OA_1}, & R \geq 0 \\ \dfrac{OC_2}{OA_2}, & R < 0 \end{cases}, \quad (34)$$

where the points B_1, B_2, C_1 and C_2 are on the upper borders of the areas and represent appropriate limiting cycles of loading . If the point A_1 or A_2 is in the area Ω_2 then the expression **(34)** will express the coefficient of the stock of fail-safe of the structure element with the crack or, that too most, stock of stability of process of gradual increase of the crack.

From geometrical reasons follows that at the given fail-safe diagram in accordance with **(33)** and **(34)** the specified coefficient of the stock expressed by the following formulas:

$$n_s = \begin{cases} \dfrac{K_{th.o}}{\left[\dfrac{2}{2+\Psi_1}(K_r + \Psi_1 K_m)\right]}, & R \geq 0 \\ \dfrac{K_{th.o}}{\left[\dfrac{2}{2+\Psi_2}(K_r + \Psi_2 K_m)\right]}, & R < 0 \end{cases}, \quad (35)$$

$$n_s = \begin{cases} \dfrac{K_{fc.o}}{\left[\dfrac{2}{2+\Psi_3}(K_r + \Psi_3 K_m)\right]}, & R \geq 0 \\ \dfrac{K_{fc.o}}{\left[\dfrac{2}{2+\Psi_4}(K_r + \Psi_4 K_m)\right]}, & R < 0 \end{cases}, \quad (36)$$

$$\Psi_1 = tg\alpha_1 = \frac{2K_{th.o}}{2K_{1c} - K_{th.o}}\ ;\ \Psi_2 = tg\alpha_2 = 2 \cdot \left(\frac{1 - K_{th.-1}}{K_{th.o}}\right);$$

$$\Psi_3 = tg\alpha_3 = \frac{2K_{fc.o}}{2K_{1c} - K_{fc.o}}\ ;\ \Psi_4 = tg\alpha_4 = 2 \cdot \left(\frac{1 - K_{fc.-1}}{K_{fc.o}}\right).$$

In the considered random process **K(t)** two-dimensional time series **(K_r,K_m)** describing the loading cycles are random and varied in accordance with growth of the crack. Therefore the values **n_s** and **n_c**, also will be random varied in time random valuations. It is possible to calculate probable characteristics of values **n_s** and **n_s** (their mean, mean square and etc.) for the given low of the probability distribution function for the system of values **(K_r,K_m)** and to receive valuations for the probabilities of events that the process **K(t)** during time **t** will not leave the border of the areas. These probabilities are parameters of reliability and are denoted as follows:

$$\begin{aligned} H_1(t) &= P\{K(\tau) \in \Omega_1\ ,\ 0 \leq \tau \leq t\} \\ H_{21}(t) &= P\{K(\tau) \in \Omega_1 + \Omega_2\ ,\ 0 \leq \tau \leq t\} \end{aligned}, \quad (37)$$

In accordance with **(30)** the value **K_p** represents probable blends of three values: zero, positive maximum and range of process **K(t)** between two next extremums **K_r=2K_a**, where **K_a** - the amplitude value **SIF** with probability density function **(26).** From here follows that the probability density function can be expressed in the form:

$$f_p(K_p) = c \cdot (\alpha \cdot \delta(K_p) + (1 - 2 \cdot \alpha) \cdot f_+(K_p) + \alpha \cdot f(K_p))\ , \quad (38)$$

where $\mathbf{C}$ - factor of standardization, $f_{+}(.), f(.)$ - probability density function of values K_{max} under $K_{min}<0$ and K_r, $\delta(.)$ - delta function, $\alpha=(k-1)/(2k)$ - standardization factor.

In accordance with **(31)** and in view of that the meanings of process for the points of trajectory inflection is identified with the mean meaning of loading cycle following expression for probability density function K_m is received:

$$f_m(K_m) = \alpha \cdot \delta(K_m) + (1-\alpha) \cdot f_i(K_m), \quad (39)$$

where f_i - probability density function for the process $K(t)$ in the points of the trajectory inflection.

The expressions **(36)** and **(39)** express the mutual probability density function $f(K_r K_m)$ for the system of values K_r and K_m by any time moment for the given crack length $l(t)$ by this time moment. The change of this density in time is coursed by gradual propagation of the crack length of which changes pursuant to the equation **(32)** where the effective value of **SIF** is defined from the expression **(36)** as:

$$K_e = \begin{cases} \dfrac{2}{2+\Psi_1}\left(K_r + \Psi_1 K_m\right), & R \geq 0 \\ \dfrac{2}{2-\Psi_2}\left(K_r + \Psi_2 K_m\right), & R < 0 \end{cases}, \quad (35)$$

Integrating the equation **(32)** in view of **(40)** gives the crack length by a time moment **t** at first and the corresponding probability density functions $f(K_r, K_m)$ and $f(K_e)$ then. It is possible to calculate the probability of event that non-stationary process $K(t)$ for time **t** will not exceed a dangerous level K_*, i.e. the probability that destruction will not take place during time t from the formula **(3):**

$$F_*(K_*, t) = 1 - \frac{1}{t_c}\int_0^t \left\{1 - F_\tau(K_*)\right\} d\tau, \quad (41)$$

where t_c - the mean interval of time between loading, $F_\Sigma(K)$ - probability function for K_e a time moment τ.

The probability meaning determined from the formula **(41)** is the main parameter of reliability of the structure's element with propagation crack under random loading processes complex by a structure.

4. Conclusion.

Given investigation was done due to special contract SPC-95-4011 and is devoted to calculate methods for fatigue longevity and survivability prediction cracked constructions under random process of loading development. Performing of this investigation is caused by growing requirements to calculate accuracy of new machines, devices and constructions reliability and service life, which are working under random loading. Used in this work methods are based on modern mechanics of materials destruction, theory of random processes and statistic dynamics of mechanical systems. The main attention in this report is devoted for the development of the random processes theory and statistic dynamics of machines directed on obtaining supplementary information about processes of loading structure essential for more accurate calculation of reliability factors, defined by sudden refusal, accumulates fatigue damages and crack growth.

The main new scientific results we had obtained with this project are follows:

1. Theory and investigate methods for obtaining of more over numeric information about random processes then in classic statistic dynamics [1 , 2] are developed. This addition information is in analysis of random process structure peculiarities which indexes have influence on accuracy of numeric estimation of reliability and service life.

2. Discovery of all random stationary process cycles given (after solving of corresponding statistic dynamic problem) by correlation function or spectral density is proposed.

3. Algorithm is developed and due to it we obtained estimations of all preceding process loading cycles influence on crack growth speed. This allows us to predict more accurately remaining (after crack appeared) construction service life.

4. Discovery of all cycles in random process of loading with complex structure allow us to use during calculation of fatigue life, survivability of construction elements with cracks traditional standardized by international standards information about fatigue resistance and crack growth characteristics.

5. The existent now service life predict methods are limited by stationary random processes in linear stationary mechanical systems. After we carried out given research we can offer the continuation of our cooperation in the next trends:

a) Development of international standards of constructions reliability and service life estimation under random processes of loading with complex structure;

b) Development of structural analysis methods of not Gaussian and non-stationary random processes in nonlinear dynamic systems;

c) Development of methods to obtain initial data for structural analysis of random processes with incomplete information about loadings and dynamic system parameters;

d) development of analyze methods of non-deterministic by structure dynamic systems;

e) Development of methods of system reliability prediction in chaotic vibrations.

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2. Svetlitsky V.V. Random vibration of mechanical systems.Moscow,1991.
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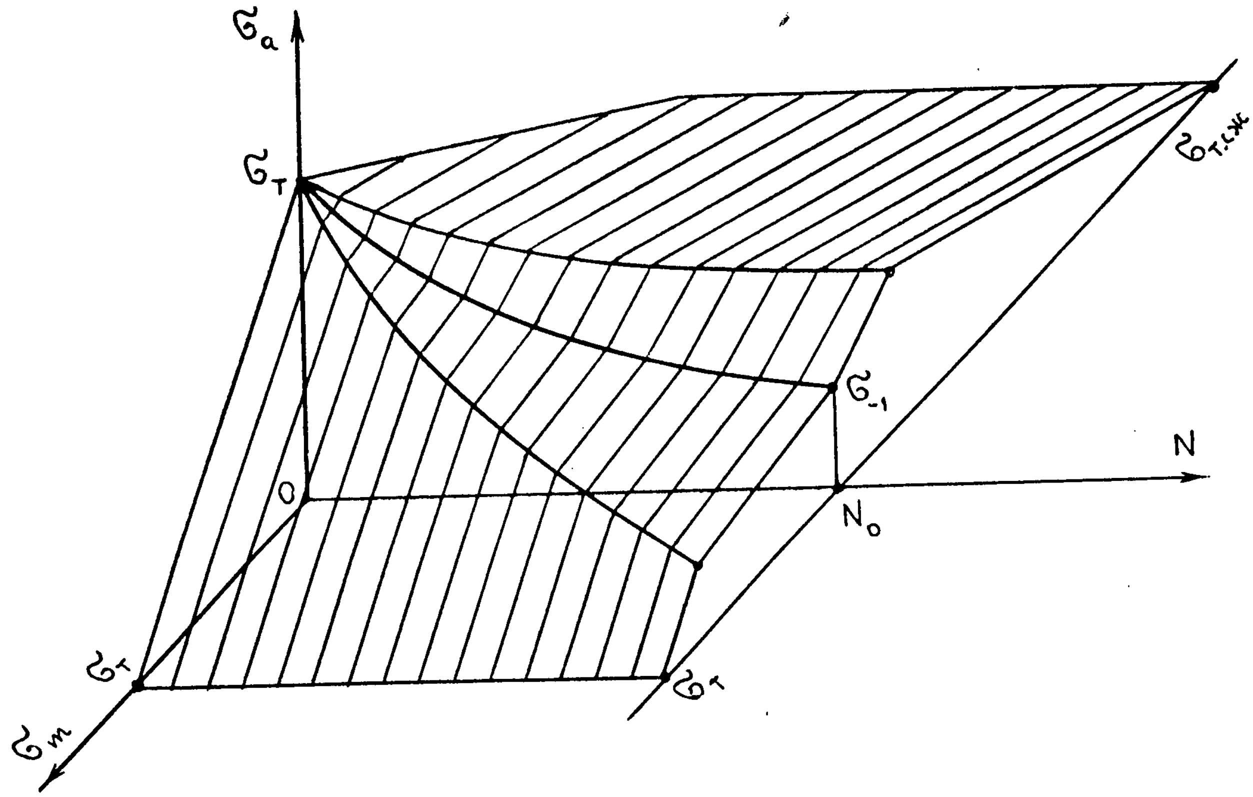

Fig. 1 Fatigue surface.

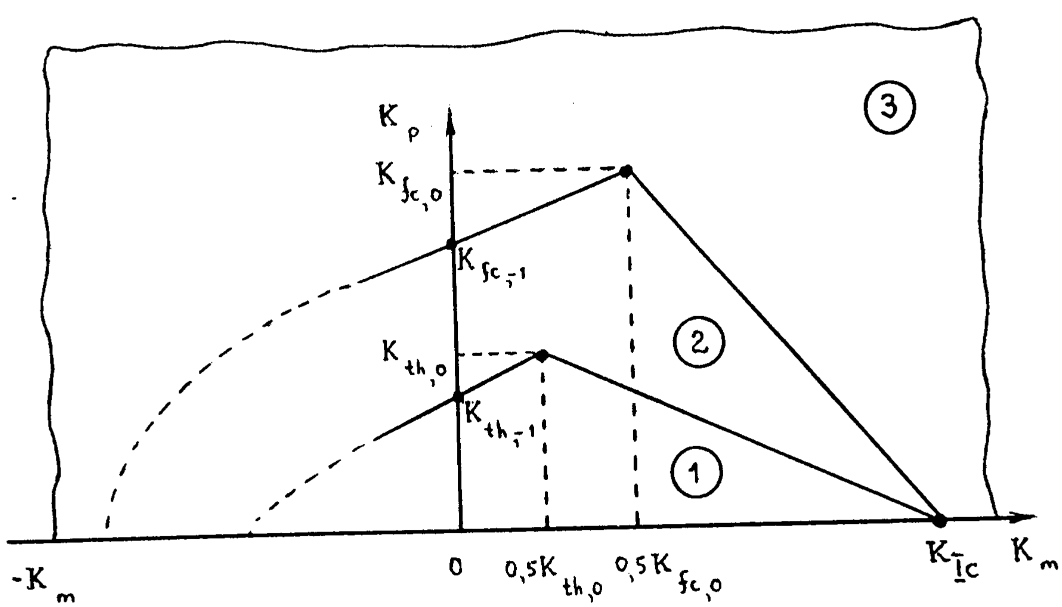

Fig. 2 Full survivability diagram.

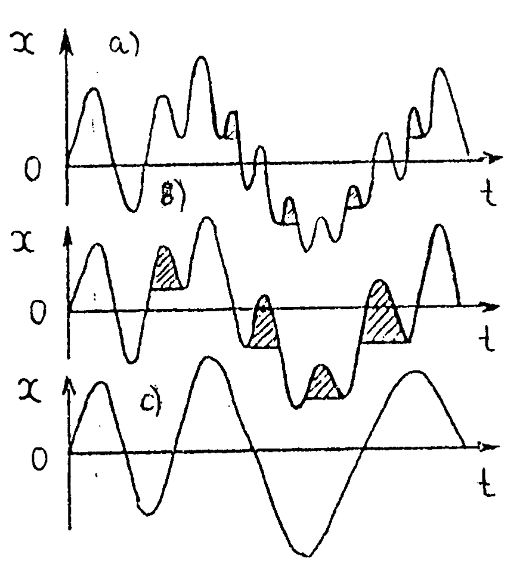

Fig.3. Transforming of complex structure a process with gradual ruling out intermediate cycles

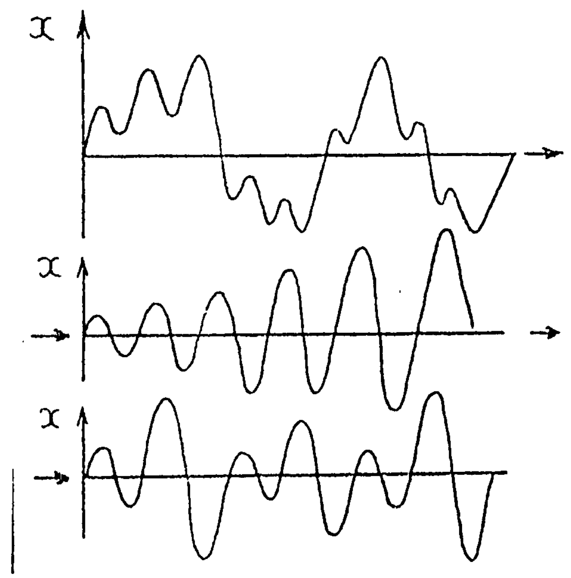

Fig.4. Transition from a process of complex structure to a process of simple structure with successive mixing of cycles.

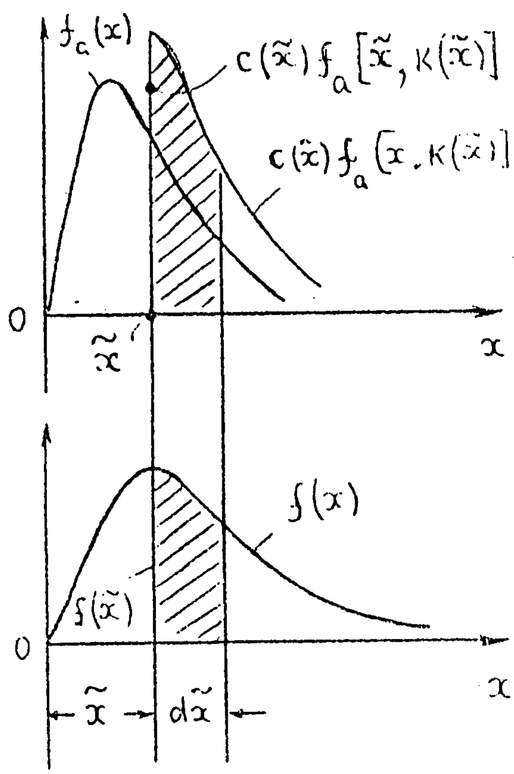

Fig.5. Forming probability distribution density for cycles amplitudes

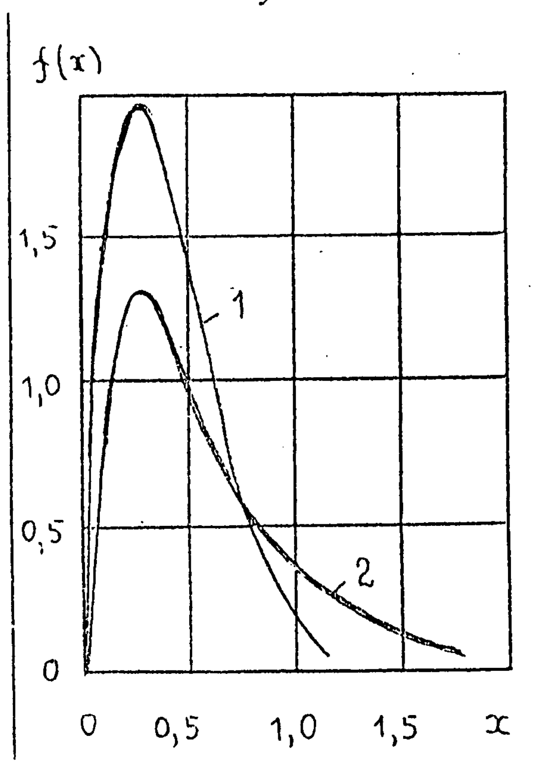

Fig.6. Probability distribution densities for amplitudes for ***K_0=3; 1*** *- initial process;* ***2*** *- process being formed.*